

Hall effect for indirect excitons in non-homogeneous magnetic field

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We study the effect of non-homogeneous out-of-plane magnetic field on the behaviour of 2D spatially indirect excitons. Due to the difference of magnetic field acting on electrons and holes the total Lorentz force affecting the center of mass motion of an indirect exciton appears. Consequently, an indirect exciton acquires an effective charge proportional to the gradient of the magnetic field. The appearance of the Lorentz force causes the Hall effect for neutral bosons which can be detected by measurement of the spatially inhomogeneous blueshift of the photoluminescence using counter-flow experiment.

Introduction. The interaction of electrically charged particles with external magnetic field leads to variety of phenomena in the condensed matter physics. The classical examples are integer and fractional quantum Hall effects extensively studied both experimentally and theoretically in last decades [1–3]. In the domain of cold atoms magnetic field can also change the behavior of the system, *e.g.* driving the BEC-BCS transition by means of Feshbach resonances [4]. However, as cold atoms are neutral objects, the application of the magnetic field does not lead to appearance of the Lorentz force and Hall effect. Meanwhile, the possibility of the generation of artificial magnetic field in atomic systems was proposed [5]. This phenomenon is based on the effect of the geometric (Berry) phase and requires illumination of the sample by several laser beams tuned in resonance with atomic transitions [6]. This has opened the way for the possibility of the observation of the analog of quantum Hall effect for neutral cold bosons and fermions.

In the field of condensed matter physics there exist as well electrically neutral excitations of the bosonic type. These are excitons – bounded electron-hole pairs. The impact of excitons onto optical and transport properties of semiconductor materials were studied intensively [7], but possibility of the Bose-Einstein condensation of direct excitons remains an open question [8]. A great step forward was achieved by using the effect of strong exciton-photon coupling in semiconductor microcavities [9]. The hybrid light-matter quasiparticles called exciton-polaritons revealed intriguing physical properties and formation of macroscopically coherent state of polaritons was experimentally reported [10]. However, the question how it is connected with standard BEC picture is still under debate [11]. Indeed, cavity polaritons have a very short lifetime (not exceeding tens of picoseconds) which prevents the possibility of full thermalization of the system.

Recently, another candidate was proposed for the achievement of BEC in the condensed matter systems.

Those are indirect excitons, the composite quasiparticles consisting of electrons and holes located in a spatially separated quantum wells and bound together by Coulomb attraction (see sketch in the Fig. 1). They obey bosonic statistics and can undergo Bose-Einstein condensation at temperatures around 1 K [12–14]. They differ from the usual 3D or 2D direct excitons by a much longer lifetime, robustness with respect to the effects of disorder and much stronger exciton-exciton interactions provided by dipole-dipole repulsion.

In this Letter we study the effect of non-homogeneous out-of-plane magnetic field on the behaviour of indirect excitons. While homogeneous magnetic field in z direction only changes the binding energy of the exciton, the non-homogeneous magnetic field acts differently on electrons and holes and results into appearance of the Lorentz force acting on center of mass (CM) of a neutral exciton. Consequently, the exciton acquires an effective “charge” and observation of an analog of the Hall effect becomes possible.

Exciton in non-homogeneous magnetic field. We start with consideration of the quantum problem of single indirect exciton in external magnetic field [15–18]. The generic quantum Hamiltonian of an electron and a hole in different quantum wells separated by distance L with external magnetic field pointed in z -direction perpendicular to the QW plane reads

$$\hat{H} = \frac{\hbar^2}{2m_e} \left(-i\nabla_e + \frac{e}{c} \mathbf{A}_e(\mathbf{r}_e) \right)^2 + \frac{\hbar^2}{2m_h} \left(-i\nabla_h - \frac{e}{c} \mathbf{A}_h(\mathbf{r}_h) \right)^2 - \frac{e^2}{\varepsilon \sqrt{(\mathbf{r}_e - \mathbf{r}_h)^2 + L^2}}, \quad (1)$$

where $\mathbf{r}_{e,h}$ are radius-vectors of 2D electrons and holes, m_e and m_h are their effective masses and L is a separation between QWs. In the further consideration we restrict ourselves to the approximation of narrow QWs. Taking into account the finite width of QWs is straightforward but requires additional computational efforts

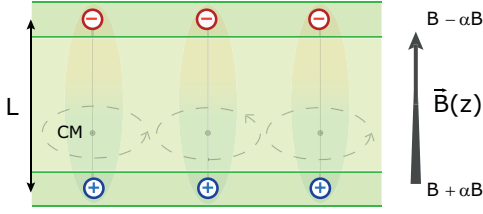


FIG. 1: (color online). Sketch of the geometry of the system. Electrons and holes located in two QWs separated by the distance L form spatially indirect excitons. The non-homogeneous in z direction magnetic field $\mathbf{B}(z)$ affects the orbital motion of the center-of-mass (CM) of exciton.

[19]. The external magnetic field acting on an electron and a hole is introduced into the Hamiltonian via vector potential $\mathbf{A}(\mathbf{r})$. The magnetic field is considered to be non-homogeneous in z direction and therefore vector potentials for electrons and holes $\mathbf{A}_e(\mathbf{r}_e)$, $\mathbf{A}_h(\mathbf{r}_h)$ are different.

The following Hamiltonian for electron-hole pair with attraction can be rewritten in terms of center-of-mass (CM) coordinates $\mathbf{R} = \beta_e \mathbf{r}_e + \beta_h \mathbf{r}_h$ and relative motion coordinates $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$, where $\beta_{e,h} = m_{e,h}/M$ and $M = m_e + m_h$. Thus, Eq. (1) can be recast as

$$\hat{H} = \frac{\hbar^2}{2m_e} \left(-i\beta_e \nabla_R - i\nabla_r + \frac{e}{c} \mathbf{A}_e(\mathbf{R} + \beta_h \mathbf{r}) \right)^2 + \quad (2)$$

$$+ \frac{\hbar^2}{2m_h} \left(-i\beta_h \nabla_R + i\nabla_r - \frac{e}{c} \mathbf{A}_h(\mathbf{R} - \beta_e \mathbf{r}) \right)^2 - \frac{e^2}{\varepsilon \sqrt{r^2 + L^2}},$$

where $\nabla_{R,r}$ are vector differential operators for CM and relative exciton motion, respectively.

We consider the case of a magnetic field $\mathbf{B}(z)$ that is non-homogeneous in z direction. Within the approximation of narrow QWs this means that 2D electrons and holes in parallel layers feel different values of B . In the following derivation we use symmetric gauge for vector potentials $\mathbf{A}_i(\mathbf{r}) = [\mathbf{B}_i \times \mathbf{r}]/2$ with values of magnetic field taken different for the electrons and the holes, $B_e = B - \Delta B/2 = B(1 - \alpha)$ and $B_h = B + \Delta B/2 = B(1 + \alpha)$, where the parameter α describes the degree of the non-homogeneity of the magnetic field, $\alpha \approx (L/2B)dB/dz$. The difference in magnetic fields acting on electrons and holes ΔB can be achieved *e.g.* by using magnetic semiconductor material for creation of the QW with electrons.

It is convenient to rewrite the Hamiltonian (1) using the phase shift transformation of the wavefunction of the exciton similar to those used for the case of the homogeneous magnetic field [17, 20]

$$\Psi'(\mathbf{R}, \mathbf{r}) = \exp \left(-i \frac{e}{2\hbar c} [\mathbf{B} \times \mathbf{r}] \cdot \mathbf{R} \right) \Psi(\mathbf{R}, \mathbf{r}). \quad (3)$$

The new Hamiltonian reads

$$\begin{aligned} \hat{H} = & -\frac{\hbar^2 \nabla_R^2}{2M} - \frac{\hbar^2 \nabla_r^2}{2\mu} + \frac{e^2 B^2 r^2}{8\mu c^2} (1 - 2\alpha\gamma + \alpha^2 \xi_3) + \frac{e\hbar}{Mc} (1 - \alpha\gamma/2) [\mathbf{B} \times \mathbf{r}] \cdot (-i\nabla_R) + \\ & + \frac{e\hbar}{2\mu c} (\gamma - \alpha\xi_2) [\mathbf{B} \times \mathbf{r}] \cdot (-i\nabla_r) + \alpha^2 \frac{e^2 B^2 R^2}{8\mu c^2} - \alpha \frac{e\hbar}{Mc} [\mathbf{B} \times \mathbf{R}] \cdot (-i\nabla_R) - \\ & - \alpha \frac{e\hbar}{2\mu c} [\mathbf{B} \times \mathbf{R}] \cdot (-i\nabla_r) - \alpha \frac{e^2}{4\mu c^2} (1 - \alpha\gamma) [\mathbf{B} \times \mathbf{R}] \cdot [\mathbf{B} \times \mathbf{r}] - \frac{e^2}{\varepsilon \sqrt{r^2 + L^2}}, \end{aligned} \quad (4)$$

where $\mu = m_e m_h / M$, $\gamma = (m_h - m_e) / M$, $\xi_3 = (m_e^3 + m_h^3) / M^3$, $\xi_2 = (m_e^2 + m_h^2) / M^2$. Note, that in the case of a homogeneous magnetic field, when $\alpha = 0$, the phase transformation (3) fully removes the action of the magnetic field on the CM of the exciton. However, in our case $B_e \neq B_h$, $\alpha \neq 0$ and one sees that the magnetic field affects the motion of the center of mass. Moreover, strictly speaking it becomes impossible to separate relative and CM motions, and numerical treatment becomes necessary.

In the Eq. (4) the first two terms correspond to the kinetic energy of the CM and relative motion of an exciton.

The third term is simply a modified diamagnetic shift, the fourth term is quasielectric field term which pushes the electron and the hole in opposite directions and the fifth term is a modified Zeeman-like term for the relative motion. All these terms exist for the homogeneous case ($\alpha = 0$) [7, 15–17].

The new sixth, seventh, eighth and ninth terms (where α enters in the overall pre-factors) describe the effect of the magnetic field on CM motion of exciton. Finally, the tenth term represents the Coulomb attraction between the electron and the hole and is responsible for the creation of the bound exciton state.

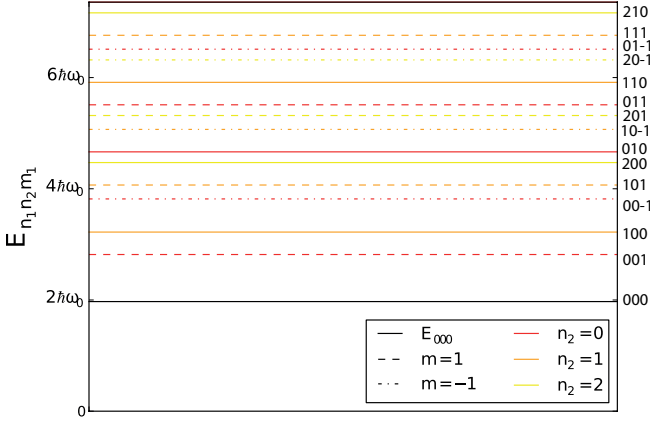


FIG. 2: (color online). The energy spectrum of the CM motion of a bound electron-hole pair in the non-homogeneous magnetic field given by Eq. (9).

Let us now examine the Hamiltonian given by Eq. (4) in more details. The terms corresponding to the exciton in a uniform magnetic field have been analyzed before. In small fields the binding energy of the exciton increases, while strong magnetic fields on the contrary lead to transition to the magnetoexciton regime [19, 21, 22]. The crossover between these two regimes is well studied. In the present paper we will focus on the additional terms which appear due to gradient of magnetic field in the z direction. As well, we do not consider the terms corresponding to the mixing of the center of mass and relative motion. The latter assumption can be justified by using the Born-Oppenheimer approximation. Indeed, considering the relative motion as fast and CM motion as slow, and averaging over fast motion it is easy to see that those terms cancel as $\langle \nabla_r \rangle = \langle [\mathbf{B} \times \mathbf{r}] \rangle = 0$.

The part of the Hamiltonian corresponding to CM motion thus reads (only underlined terms in the Eq. (4) are considered)

$$\hat{H} = \frac{\hbar^2}{2m_e} (-i\beta_e \nabla_R - \alpha \frac{e}{\hbar c} \mathbf{A}(\mathbf{R}))^2 + \quad (5)$$

$$+ \frac{\hbar^2}{2m_h} (-i\beta_h \nabla_R - \alpha \frac{e}{\hbar c} \mathbf{A}(\mathbf{R}))^2.$$

which after simple algebra yields

$$\hat{H} = \frac{1}{2M} \left(\hat{\mathbf{P}} - \frac{e^*}{c} \mathbf{A}(\mathbf{R}) \right)^2 + \frac{e^{*2} B^2 R^2 \gamma^2}{8c^2 \mu}, \quad (6)$$

where $\hat{\mathbf{P}} = -i\hbar \nabla_R$ is a CM momentum of the electron-hole pair, $\mathbf{A}(\mathbf{R}) = [\mathbf{B} \times \mathbf{R}]/2$. One can see that expression (6) looks exactly the same as the Hamiltonian of a 2D particle with effective charge e^* placed in a magnetic field perpendicular to the plain in presence of an in-plane external parabolic confining potential $U = e^{*2} B^2 R^2 \gamma^2 / 8c^2 \mu$. The value of the effective charge of the indirect exciton which describes its response to

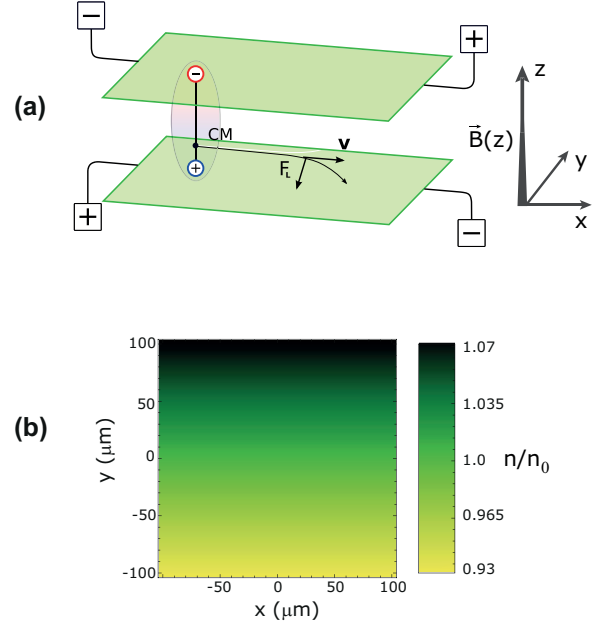


FIG. 3: (color online). (a) Geometry of the counterflow experiment. Applying the voltage of different polarities for the electron and hole subsystems leads to the generation of the flux of indirect excitons in x direction. (b) The profile of 2D density of indirect excitons in the non-homogeneous magnetic field. The drift velocity is taken $V = 10^4$ cm/s, magnetic field difference $\Delta B = 1$ mT and background concentration $n_0 = 10^9$ cm $^{-2}$. The Hall effect causes deviation of exciton density equal of about 7%.

inhomogeneous magnetic field is

$$e^* = 2\alpha e = \frac{\Delta B}{B} e. \quad (7)$$

As one can expect, it is proportional to the difference of the magnetic field acting on the electron and hole subsystems and vanishes for the uniform case. Note that this effective charge describes only the response to the magnetic field and external electric field will still not affect the motion of an exciton which remains a neutral particle.

The solution of the eigenvalue problem for a particle in a magnetic field in presence of parabolic confined potential is well known [23]. This allows us to write the expression for the energy spectrum of an indirect exciton in a non-homogeneous magnetic field as

$$E_{n_1 n_2 m_1} = \hbar \sqrt{\tilde{\omega}^2 + \omega_0^2} \left(n_1 + \frac{1}{2} + \frac{|m_1|}{2} \right) + \quad (8)$$

$$+ \frac{\hbar \omega_0}{2} \left(n_2 + \frac{1}{2} \right) - \frac{\hbar \omega_0}{2} m_1,$$

where n_1 , n_2 and m_1 are integer numbers. The quantum number m_1 corresponds to the angular momentum. The cyclotron frequency ω_0 and the potential well quantiza-

tion frequency $\tilde{\omega}$ read

$$\omega_0 = \frac{e^* B}{Mc} = \frac{e \cdot \Delta B}{Mc} = \frac{\mu}{m_h - m_e} \tilde{\omega}. \quad (9)$$

The energy levels given by Eq. (9) are shown in the Fig. 2. Note, that differently from the case of the Landau quantization for the free electrons, the spectrum is not equidistant. As well, it should be noted that the energy distance between the levels corresponding to the quantization of CM motion of an exciton is much smaller than the distance between excitonic levels appearing due to the quantization of relative motion. The spectrum in the Fig. 2 can be thus interpreted as "fine structure" of 1s exciton state in non-homogeneous magnetic field.

For $\Delta B = 1$ mT we can estimate $\hbar\omega_0 = 0.2$ μ eV, which is much smaller than the thermal energy corresponding to 1 K. Therefore direct observation of quantization of the energy levels is hardly possible. One thus needs to propose another way of the observation of the effect of non-homogeneous magnetic field on indirect excitons.

We believe that it can be done analogically to the detection of the classical Hall effect (see Fig. 3). The key point will be to organize a flow of excitons, which is a more complicated task than for the case of the electrons since excitons are electrically neutral particles and application of an electric field will not lead to the appearance of the exciton current. However, this current can be created in a counter-flow experiment, where polarity of the drain-source voltage is different for the layers of the electrons and the holes. The counterflow technique is a well developed method and was widely used by Eisenstein and co-workers for the study of the physical properties of Quantum Hall bilayers [24].

The drift velocity of an indirect exciton in counter-flow experiment will be governed by the value of the electric fields in electron and hole layers $E_{e,h}$ and mobilities of the electrons and the holes $\mu_{e,h}$:

$$\mathbf{V} = \frac{\mu_e \mu_h}{\mu_e + \mu_h} (\mathbf{E}_h - \mathbf{E}_e). \quad (10)$$

One should note that the velocity can not be too large since magnetic field tends to unbind fast moving excitons [7, 19].

Due to non-zero effective charge of the indirect exciton the Lorentz force acting on CM coordinates appears $\mathbf{F}_L = e^* \mathbf{V} \times \mathbf{B}/c$. It deviates the moving exciton in the direction perpendicular to the direction of electric field $(\mathbf{E}_h - \mathbf{E}_e)$. In the stationary regime this force should be compensated by the force provided by a gradient of potential energy of the exciton-exciton repulsion, which can be estimated as $\mathbf{F}_p = -U_0 \nabla n(\mathbf{r})$, with $U_0 \approx 4\pi e^2 L/\varepsilon$, where L is the distance between the QWs with the electrons and holes and ε is a dielectric permittivity. Putting $\mathbf{F}_L = \mathbf{F}_p$ one can determine the spatial profile of the exciton concentration in the Hall bar geometry shown in the

Fig. 3, where the flux of the excitons is directed along x axis,

$$n(y) = n_0 + \frac{eV\Delta B}{U_0} \frac{y}{2}, \quad (11)$$

and n_0 denotes a background 2D density of the indirect excitons. The density profile is shown in the Fig. 3 for the magnetic field $\Delta B = 1$ mT, $n_0 = 10^9$ cm^{-2} and drift velocity $V = 10^4$ cm/s . The variation of the density can be measured by analyzing the spatial variation of the blueshift of the exciton photoluminescence in the near-field experiment.

Conclusions. In conclusion, we proposed a method of generation of the effective magnetic field acting on a system of electrically neutral spatially indirect excitons. We analyzed the energy spectrum of the system and predicted the existence of an analog of the classical Hall effect which can be detected in a counter-flow experiment.

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- [1] K. von Klitzing, G. Dorda, M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
 - [2] D. C. Tsui, H. L. Stormer, A. C. Gossard, **48**, 1559 (1982); H. L. Stormer, A. Chang, D. C. Tsui, H. C. M. Hwang, A. C. Gossard, W. Wiegman, Phys. Rev. Lett. **50**, 1953 (1983).
 - [3] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
 - [4] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).
 - [5] J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg, Rev. Mod. Phys. **83**, 1523 (2011).
 - [6] Y.-J. Lin, R. L. Compton, K. Jimenez-Garcia, J. V. Porto and I. B. Spielman, Nature **462**, 628 (2009).
 - [7] R. S. Knox, *Theory of Excitons* (Academic Press Inc., 1963).
 - [8] S. A. Moskalenko and D. W. Snoke, *Bose-Einstein Condensation of Excitons and Biexcitons* (Cambridge University Press, Cambridge, U.K., 2000).
 - [9] A. V. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, Oxford, 2007).
 - [10] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymanska, R. Andre, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and Le Si Dang, Nature **443**, 409 (2006).
 - [11] L. V. Butov, A. V. Kavokin, Nature Photonics **6**, 2 (2011).
 - [12] L. V. Butov, J. Phys.: Condens. Matter **19**, 295202 (2007).
 - [13] D. Snoke, Science **298**, 1368 (2002); D. W. Snoke, Y. Liu, Z. Vrs, L. Pfeiffer, K. West. Solid State Comm. **134**, 37 (2005).
 - [14] L. V. Butov, C. W. Lai, A. L. Ivanov, A. C. Gossard, and D. S. Chemla, Nature **417**, 47 (2002); L. V. Butov, A. C. Gossard, and D. S. Chemla, Nature **418**, 751 (2002).

- [15] R. T. Elliot, R. Loudon, J. Phys. Chem. Solids **15**, 196 (1960).
- [16] H. Hasegawa, R. E. Howard, J. Phys. Chem. Solids. **21**, 179 (1961).
- [17] L. P. Gor'kov, I. E. Dzyaloshinskii, Sov. Phys. JETP **26**, 449 (1968).
- [18] W. Edelstein, H. N. Spector, and R. Marasas, Phys. Rev. B **39**, 7697 (1989).
- [19] Yu. E. Lozovik, I. V. Ovchinnikov, S. Yu. Volkov, L. V. Butov, and D. S. Chemla, Phys. Rev. B **65**, 235304 (2002).
- [20] J. A. K. Freire, A. Matulis, F. M. Peeters, V. N. Freire and G. A. Farias, Phys. Rev. B **61**, 2895 (2000).
- [21] S. A. Moskalenko, M. A. Liberman, D. W. Snoke, V. V. Boan, Phys. Rev. B **66**, 245316 (2002); S. A. Moskalenko, M. A. Liberman, D. W. Snoke, V. V. Boan, B. Johansson, Physica E **19**, 278 (2003).
- [22] Yu. E. Lozovik and A. M. Ruvinsky, Phys. Lett. A **227**, 271 (1997).
- [23] V. I. Kogan, V. M. Galitsky, *Problems in Quantum Mechanics* (Englewood Cliffs, N.J.: Prentice-Hall, 1963).
- [24] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. *84*, 5808 (2000); J. Eisenstein, Science **305**, 950 (2004).